

Reg. No. : .....

Name : .....

**Combined First and Second Semester B.Tech. Degree Examination,  
April 2014  
(2013 Scheme)**

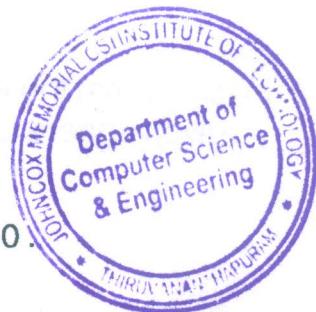
**13.101 – ENGINEERING MATHEMATICS – I (ABCEFHMNPRSTU)**

Time : 3 Hours

Max. Marks : 100

**PART – A**

Answer **all** questions. **Each** question carries 4 marks.



1. If  $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$ , show that  $(1-x^2)y_{n+1} - (2n+1)xy_n - n^2y_{n-1} = 0$ .
2. If  $u = \log\left(\frac{x^4+y^4}{x+y}\right)$  show that  $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$ .
3. Evaluate  $\iint_R e^{2x+3y} dxdy$  over the triangle bounded by  $x = 0$ ,  $y = 0$  and  $x + y = 1$ .
4. Find the inverse Laplace Transform of  $\frac{1}{s(s+1)^3}$ .
5. Prove that the inverse of an orthogonal matrix is orthogonal and its transpose is also orthogonal.

**PART – B**

Answer **one full question from each Module**. **Each** question carry 20 marks.

**Module – 1**

6. a) Show that the evolute of the tractrix  $x = a(\cos t + \log \tan t/2)$ ,  $y = a \sin t$  is the catenary  $y = a \cosh \frac{x}{a}$ .

b) If  $x = e^{r\cos\theta} \cdot \cos(r\sin\theta)$  and  $y = e^{r\cos\theta} \sin(r\sin\theta)$ , prove that  $\frac{\partial x}{\partial r} = \frac{1}{r} \frac{\partial y}{\partial \theta}$ ,

$$\frac{\partial y}{\partial r} = -\frac{1}{r} \frac{\partial x}{\partial \theta} \text{ and hence deduce that } \frac{\partial^2 x}{\partial r^2} + \frac{1}{r} \frac{\partial x}{\partial r} + \frac{1}{r^2} \frac{\partial^2 x}{\partial \theta^2} = 0.$$

7. a) Evaluate  $\lim_{x \rightarrow 0} \left( \frac{\tan x}{x} \right)^{\frac{1}{x^2}}$ .

b) Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.

### Module – 2

8. a) Evaluate  $\iint \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}} dx dy$  over +ve quadrant of the circle  $x^2 + y^2 = 1$  by changing in to polar coordinates.

b) Find the volume of the ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ .

9. a) Evaluate  $\int_0^1 \int_{4y}^4 e^{x^2} dx dy$  by changing the order of Integration.

b) Find the area lying between the parabola  $y = 4x - x^2$  and the line  $y = x$ .

### Module – 3

10. a) If  $L\left(\frac{\sin t}{t}\right) = \tan^{-1}\left(\frac{1}{s}\right)$ . Find  $L\left(\frac{\sin at}{t}\right)$ .

b) Solve  $\frac{d^2y}{dx^2} - 4y = x \sinh x$ .

11. a) Apply convolution theorem to evaluate  $L^{-1} \frac{s}{(s^2 + a^2)^2}$ .

b) Solve the equation  $y'' - 3y' + 2y = 4t + e^{3t}$  given  $y(0) = 1, y'(0) = -1$ .

### Module - 4

12. a) Solve  $5x - 2y + z = 4, 7x + y - 5z = 8, 3x + 7y + 4z = 10$  using LU factorization method.

b) Diagonalise the matrix  $\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$  and obtain the modal matrix.

13. a) Find  $A^{-1}$  using Cayley Hamilton theorem for the matrix  $\begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$ .

b) Reduce  $3x^2 + 3z^2 + 4xy + 8xz + 8yz$  in to canonical form.

